Baryon Modes of B Meson Decays

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Abstract

The baryon decay modes of $B, \bar{B} \to N_1\bar{N}_2(f)$, $\bar{N}_1 N_2(\bar{f})$ provide a frame work to test CP-invariance in baryon sector. It is shown that in the rest frame of B, N_1 and $\bar{N}_2$ come out with longitudnal polarization $\lambda_1 = \lambda_2 = \pm 1$ with decay width $\Gamma_f = \Gamma_f^{++} + \Gamma_f^{--}$ and the asymmetry parameter $\alpha_f = \Delta\Gamma_f = \Gamma_f^{++} - \Gamma_f^{--}$. It is shown that CP invariance prediction $\alpha_f = -\bar{\alpha}_{\bar{f}}$ can be tested in these decay modes; especially in the time dependent decays of $B_q^0 - \bar{B}_q^0$ complex. Apart from this, it is shown that decay modes $B(\bar{B}) \to N_1\bar{N}_2(\bar{N}_1 N_2)$ and subsequent non leptonic decays of $N_2, \bar{N}_2$ or $(N_1, \bar{N}_1)$ into hyperon (antihyperon) also provide a frame work to study CP-odd observables in hyperon decays.

1 Introduction

The CP-violation in kaon and $B_q^0 - \bar{B}_q^0$ systems has been extensively studied [1]. There is thus a need to study CP-violation outside these systems. In hyperon decays, the observables are the decay rate Γ, asymmetry parameter α, the transverse polarization β and longitudnal polarization γ [2]. CP asymmetry predicts $\bar{\Gamma} = \Gamma$, $\bar{\alpha} = -\alpha$, $\bar{\beta} = -\beta$, where these observables correspond to non-leptonic hyperon decays $N \to N'\pi$ and $\bar{N} \to \bar{N}'\bar{\pi}$. Thus

to leading order CP -odd observables are [3]

$$\delta\Gamma = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}},\ \delta\alpha = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}},\ \delta\beta = \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} \tag{1}$$

'The decays of $B(\bar{B})$ mesons to baryon-antibaryon pair $N_1\ \bar{N}_2$ ($\bar{N}_1\ N_2$) and subsequent decays of $N_2, \bar{N}_2$ or $(N_1, \bar{N}_1)$ to a lighter hyperon (antihyperon) plus a meson provide a means to study CP-odd observables as for example in the process

$$e^- e^+ \to B, \bar{B} \to N_1 \bar{N}_2 \to N_1 \bar{N}_2' \pi,\ \bar{N}_1 N_2 \to \bar{N}_1 N_2' \pi$$

Apart from the above motivation, the baryon decay modes of B-mesons are of intrinsic intrest by themselves as we discuss below. The baryon decay modes of $B_d^0 - \bar{B}_d^0$ have also been discussed in a differnt context in [4].

In the rest frame of B, N_1 and $\bar{N}_2$ come out longitudnally polarized with polarization

$$\left(\lambda_1 \equiv \frac{E_1}{m_1}\mathbf{n}\cdot\mathbf{s}_1\right) = \left(\lambda_2 \equiv \frac{E_2}{m_2}(-\mathbf{n}\cdot\mathbf{s}_2)\right) = \pm 1,$$

where

$$\begin{aligned} \mathbf{p}_1 &= |\mathbf{p}|\,\mathbf{n}, \mathbf{p}_2 = -|\mathbf{p}|\,\mathbf{n}, \mathbf{s}_1 = \frac{m_1}{E_1}\mathbf{n} \\ \mathbf{s}_2 &= -\frac{m_2}{E_2}\mathbf{n} \end{aligned}$$

s_1^μ, s_2^μ are polarization vectors of N_1 and $\bar{N}_2$ respectively ($p_1 \cdot s_1 = 0$, $p_2 \cdot s_2 = 0$, $s_1^2 = -1 = s_2^2$). The decay $B \to N_1 \bar{N}_2(f)$ is described by the matrix element

$$M_f = F_q e^{+i\phi} \left[\bar{u}(\mathbf{p}_1)(A_f + \gamma_5 B_f) v(\mathbf{p}_2)\right] \tag{2}$$

where F_q is a constant containing CKM factor, ϕ is the weak phase. The amplitude A_f and B_f are in general complex in the sense that they incorporate the final state phases δ_p^f and δ_s^f. Note that A_f is the parity violating amplitude (p-wave) whereas B_f is parity conserving amplitude (s-wave). The CPT invariance gives the matrix elements for the decay $\bar{B} \to \bar{N}_1 N_2(\bar{f})$:

$$\bar{M}_{\bar{f}} = F_q e^{-i\phi} \left[\bar{u}(\mathbf{p}_2)(-A_f + \gamma_5 B_f) v(\mathbf{p}_1)\right] \tag{3}$$

If the decays are described by a single matrix element M_f, then CPT and CP invariance give the same prediction viz

$$\bar{\Gamma}_{\bar{f}} = \Gamma_f,\ \bar{\alpha}_{\bar{f}} = -\alpha_f,\ \bar{\beta}_{\bar{f}} = -\beta_f,\ \bar{\gamma}_{\bar{f}} = \gamma_f \tag{4}$$

2 Decay Rate and Asymmetry Parameters:

The decay width for the mode $B \to N_1 \bar{N}_2(f)$ is given by

$$\begin{aligned}
\Gamma_f &= \frac{m_1 m_2}{2\pi m_B^2} |\mathbf{p}| \, |M_f|^2 \\
&= \frac{F_q^2}{2\pi m_B^2} |\mathbf{p}| \left[(p_1 \cdot p_2 - m_1 m_2) |A_f|^2 + (p_1 \cdot p_2 + m_1 m_2) |B_f|^2 \right] \qquad (5)
\end{aligned}$$

In order to take into account the polarization of N_1 and $\bar{N}_2$, we give the general expression for $|M_f|^2$

$$\begin{aligned}
|M_f|^2 &= \frac{F_q^2}{16 m_1 m_2} Tr \left[\begin{array}{c} (\not{p}_1 + m_1)(1 + \gamma_5 \gamma \cdot s_1)(A_f + \gamma_5 B_f)(\not{p}_2 - m_2) \\ \times (1 + \gamma_5 \gamma \cdot s_2)(A_f^* - \gamma_5 B_f^*) \end{array} \right] \\
&= \frac{4F_q^2}{16 m_1 m_2} \left[\begin{array}{c} |A_f|^2 (p_1 \cdot p_2 - m_1 m_2) + |B_f|^2 (p_1 \cdot p_2 + m_1 m_2) \\ -(A_f B_f^* + B_f A_f^*)(m_2 p_1 \cdot s_2 + m_1 p_2 \cdot s_1) \\ -i(A_f B_f^* - B_f A_f^*)(\epsilon^{\mu\nu\rho\lambda} p_1^\mu s_1^\nu p_2^\rho s_2^\lambda) \\ + m_1 m_2 (|A_f|^2 + |B_f|^2) s_1 \cdot s_2 \\ + \left(|A_f|^2 - |B_f|^2 \right) \left(-p_1 \cdot p_2 s_1 \cdot s_2 + (p_1 \cdot s_2)(p_2 \cdot s_1) \right) \end{array} \right] \qquad (6)
\end{aligned}$$

It is clear that Eqs. (4) follows from Eqs.(2) and (6). In the rest frame of B, we get from Eqs.(5) and (6)

$$|M_f|^2 = F_q^2 \frac{2E_1 E_2}{4 m_1 m_2} \left[\left| a_s^f \right|^2 + \left| a_p^f \right|^2 \right] \left\{ \begin{array}{c} 1 + \alpha_f \left(\frac{m_1}{E_1} \mathbf{n}_1 \cdot \mathbf{s}_1 - \frac{m_2}{E_2} \mathbf{n} \cdot \mathbf{s}_2 \right) \\ + \beta_f \mathbf{n} \cdot (\mathbf{s}_1 \times \mathbf{s}_2) + \gamma_f \left[(\mathbf{n}_1 \cdot \mathbf{s}_1)(\mathbf{n} \cdot \mathbf{s}_2) - \mathbf{s}_1 \cdot \mathbf{s}_2 \right] \\ - \frac{m_1 m_2}{E_1 E_2} (\mathbf{n} \cdot \mathbf{s}_1)(\mathbf{n} \cdot \mathbf{s}_2) \end{array} \right\} \qquad (7)$$

where

$$\begin{aligned}
a_s &= \sqrt{\frac{p_1 \cdot p_2 + m_1 m_2}{2E_1 E_2}} B, \;\; a_p = -\sqrt{\frac{p_1 \cdot p_2 - m_1 m_2}{2E_1 E_2}} A \qquad (8) \\
\alpha_f &= \frac{2 S_f P_f \cos(\delta_s^f - \delta_p^f)}{S_f^2 + P_f^2}, \;\; \beta_f = \frac{2 S_f P_f \sin(\delta_s^f - \delta_p^f)}{S_f^2 + P_f^2} \\
\gamma_f &= \frac{S_f^2 - P_f^2}{S_f^2 + P_f^2}, \;\; a_s = S_f e^{i\delta_s^f}, a_p^f = P_f e^{i\delta_p^f} \qquad (9)
\end{aligned}$$

However in the rest frame of B, due to spin conservation

$$\frac{E_1}{m_1} \mathbf{n} \cdot \mathbf{s}_1 = \frac{E_2}{m_2} (-\mathbf{n} \cdot \mathbf{s}_2) = \pm 1 \qquad (10)$$

Thus invariants multiplying β_f and γ_f vanish. Hence we have

$$
\begin{aligned}
|M_f|^2 &= \left(\frac{2E_1E_2}{m_1m_2}\right) F_q^2(S_f^2+P_f^2)\left[(1+\lambda_1\lambda_2)+\alpha_f(\lambda_1+\lambda_2)\right] && (11)\\
\Gamma_f &= \Gamma_f^{++}+\Gamma_f^{--}=\frac{2E_1E_2}{2\pi m_B^2}\,|\vec{p}|\,F_q^2\left[S_f^2+P_f^2\right]=\bar{\Gamma}_{\bar{f}} && (12)\\
\Delta\Gamma_f &= \frac{\Gamma_f^{++}-\Gamma_f^{--}}{\Gamma_f^{++}+\Gamma_f^{--}}=\alpha_f\ ;\ \Delta\bar{\Gamma}_{\bar{f}}=\bar{\alpha}_{\bar{f}}=-\alpha_f && (13)
\end{aligned}
$$

Eqs.(12) and (13) follow from CP or CPT invariance. It will be of intrest to test these equations.

In this paper, we confine ourself to decays $B\to N_1\bar{N}_2(\bar{B}\to\bar{N}_1N_2)$ described by a single matrix element M_f $(\bar{M}_{\bar{f}})$ i.e. to the effective Lagrangians

$$
\begin{aligned}
\mathcal{L} &= V_{cb}V_{uq}^*[\bar{q}u]_{V-A}[\bar{c}b]_{V-A}+h.c. && (14)\\
\mathcal{L} &= V_{ub}V_{cq}^*[\bar{q}c]_{V-A}[\bar{u}b]_{V-A}+h.c && (15)
\end{aligned}
$$

where $q=d$ or s. For the decay modes described by the above Lagrangians, there are no contributions from the penguin diagrams. The Lagrangian given in Eq.(14) is relevant for the decays

$$
\begin{aligned}
\text{i) } B_q^0 &\to N_1\bar{N}_2(f);\ \bar{B}_q^0\to\bar{N}_1N_2(\bar{f})\\
N_1N_2 &: p\Lambda_c^+,\Sigma^+\Xi_c^+,\frac{1}{\sqrt{6}}\Lambda\Xi_c^0,\frac{1}{\sqrt{2}}\Sigma^0\Xi_c^0\\
B_c^+ &\to p\bar{n},\Sigma^+\bar{\Lambda}(q=d);\ B_c^+\to p\bar{\Lambda},\ \Sigma^+\bar{\Xi}^0\ (q=s)
\end{aligned}
$$

For the decay modes(i), the weak phase $\phi=0$ and the decay matrix elements M_f and $\bar{M}_{\bar{f}}$ are given by Eqs.(2) and (3). For the Lagrangian given in Eq.(15), the relevant decay modes are

$$
\begin{aligned}
\text{ii) } \bar{B}_q^0 &\to N_1\bar{N}_2(f);\ B_q^0\to\bar{N}_1N_2(\bar{f})\\
B^- &\to N_1\bar{N}_2:\ n\bar{\Lambda}_c^-,\ \frac{1}{\sqrt{6}}\Lambda\bar{\Xi}_c^-,\ \ -\frac{1}{\sqrt{2}}\Sigma^0\bar{\Xi}_c^-,\ \Sigma^-\bar{\Xi}_c^0\ (q=d)
\end{aligned}
$$

$$
-\frac{2}{\sqrt{6}}\Lambda\bar{\Lambda}_c^-,\ \Xi^0\bar{\Xi}_c^-,\ \Xi^-\bar{\Xi}_c^0\ (q=s)
$$

For various decay channels (i) and (ii), we have explicitly shown the $SU(3)$ factors. For the decay modes (ii), the weak phase $\phi=\phi_3/\gamma$, which arises

from $V_{ub} = |V_{ub}|\, e^{-i\gamma}$. For the decay modes (ii), the matrix elements $\bar{M}'_f$ and $M'_{\bar{f}}$ are given by

$$\bar{M}'_f = e^{-i\phi_3} F'_q [\bar{u}(\mathbf{p}_1)(A'_{\bar{f}} + \gamma_5 B'_{\bar{f}}) v(\mathbf{p}_2)] \tag{16}$$

$$M'_{\bar{f}} = e^{i\phi_3} F'_q [\bar{u}(\mathbf{p}_2)(-A'_{\bar{f}} + \gamma_5 B'_{\bar{f}}) v(\mathbf{p}_1)] \tag{17}$$

Hence the decay widths and $CP-$asymmetry parameters are given by

$$\bar{\Gamma}'_f = \Gamma'_{\bar{f}} = \frac{2E_1E_2}{8\pi m_B^2} |\mathbf{p}|\, F'^2_q (S'^2_{\bar{f}} + P'^2_{\bar{f}}) \tag{18}$$

$$\bar{\alpha}'_f = -\alpha'_{\bar{f}} = \frac{2S'_{\bar{f}} P'_{\bar{f}} \cos(\delta^{\bar{f}}_s - \delta^{\bar{f}}_p)}{(S'^2_{\bar{f}} + P'^2_{\bar{f}})} \tag{19}$$

Now

$$F_q = \frac{G_F}{\sqrt{2}} (a_2, a_1) V_{cb} V_{uq} \tag{20}$$

$$F'_q = \frac{G_F}{\sqrt{2}} (a_2, a_1) |V_{ub}|\, V_{cq} \tag{21}$$

Define

$$\begin{aligned} r &= \frac{F'_q}{F_q} = \frac{|V_{ub}|\, V_{cq}}{V_{cb} V_{uq}} = -\lambda^2 \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \text{ for } q = d \\ &= \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \text{ for } q = s \end{aligned} \tag{22}$$

a_2 (a_1) are factors which account for color supressed (without color supressed) matrix elements. From Eqs.(12), (20), we get

$$\begin{aligned} \frac{\Gamma(B^0_s \to p\bar{\Lambda}^-_c)}{\Gamma(B^0_d \to p\bar{\Lambda}^-_c)} &= \lambda^2 \left(\frac{m_{B_d}}{m_{B_s}}\right)^2 \frac{[E_1E_2\,|\vec{p}|]_{B_s}}{[E_1E_2\,|\vec{p}|]_{B_d}} \xi^2 \\ &\approx \lambda^2 \xi^2 \end{aligned} \tag{23}$$

where ξ is a measure of $SU(3)$ violation.

Now B^0_q, $\bar{B}^0_q$ annihilate into baryon-antibaryon pair $N_1\bar{N}_2$ through W-exchange as depicted in Figs (1a) and (1b). $B^- \to N_1\bar{N}_2$ through annihilation diagram is shown in Fig (2). It is clear from Fig (1a) and (1b), that we have the same final state configuration for $B^0_q, \bar{B}^0_q \to N_1\bar{N}_2$. Thus one would expect

$$\begin{aligned} S'_{\bar{f}} &= S_f,\ P'_{\bar{f}} = P_f \\ \delta^{\bar{f}}_s &= \delta^f_s, \delta^{\bar{f}}_p = \delta^f_p \end{aligned} \tag{24}$$

Hence we have

$$
\begin{aligned}
\Gamma'_{\bar{f}} &= \bar{\Gamma}'_f = r^2 \Gamma_f && (25)\\
\bar{\alpha}'_f &= -\alpha'_{\bar{f}} = \alpha_f = -\bar{\alpha}_{\bar{f}} && (26)\\
\frac{\Gamma(\bar{B}^0_s \to p\bar{\Lambda}^-_c)}{\Gamma(B^0_s \to p\bar{\Lambda}^-_c)} &= \left(\bar{\rho}^2 + \bar{\eta}^2\right) && (27)\\
\frac{\Gamma(B^- \to \Lambda\bar{\Lambda}^-_c)}{\Gamma(B^0_d \to p\bar{\Lambda}^-_c)} &\approx \frac{2}{3}\left(\lambda\frac{a_1}{a_2}\right)^2 \left(\bar{\rho}^2 + \bar{\eta}^2\right) && (28)
\end{aligned}
$$

Eq.(28) is valid in $SU(3)$ limit, but $SU(3)$ breaking effects can be taken into account by using physical masses for proton and Λ hyperon in the kinematical factors.

Above predictions can be tested in future experiments on baryon decay modes of B-mesons. In particular $\bar{\alpha}'_f = \alpha_f$ would give direct confirmation of Eqs.(24).

Finally, we discuss $B^0_d \to p\bar{\Lambda}^-_c$ decay. For this decay mode the experimental branching ratio is $(2.2 \pm 0.8) \times 10^{-5}$[5]. Using the experimental value for $\tau_{B^0_d}$, we obtain

$$
\Gamma(B^0_d \to p\bar{\Lambda}^-_c) = (9.46 \pm 3.44) \times 10^{-15} \text{ MeV} \tag{29}
$$

The decay width in terms of $[S_f^2 + P_f^2]$ is given by

$$
\Gamma_f = \frac{G_F^2}{2} \left|V_{cb}\right|^2 \left|V_{ud}\right|^2 a_2^2 \left(S_f^2 + P_f^2\right) \left[\frac{2E_1E_2}{2\pi m_B^2} \left|\mathbf{p}\right|\right] \tag{30}
$$

Using $|V_{cb}| = 41.6 \times 10^{-3}$, $|V_{ud}| = 0.97378$ [5], $a_2 = 0.226$ and noting that

$$
\frac{2E_1E_2 \left|\mathbf{p}\right|}{2m_B^2} \approx 1.01 \text{ GeV}
$$

we get

$$
\Gamma_f = [9.09 \times 10^{-25} \text{MeV}^{-3}][S_f^2 + P_f^2] \tag{31}
$$

Using Eq.(29), we get

$$
\left(S_f^2 + P_f^2\right) = (1.04 \pm 0.38) \times 10^{10} \text{MeV}^4 \tag{32}
$$

In order to express $\left(S_f^2 + P_f^2\right)$ in terms of dimensionless form factors, we use $B^- \to l^-\bar{\nu}_l$ decay as a guide, which also occurs through a diagram similar to Fig 2.

For the decay $B^- \to l^- \nu_l$,

$$\begin{aligned}\Gamma(B^- \to l^- \bar{\nu}_l) &= \frac{G_F^2}{2} \left|V_{ub}\right|^2 \left(\frac{2E_1 E_2}{2\pi m_B^2}\right) |\mathbf{p}| \left[S^2 + P^2\right] \\ &= \frac{G_F^2}{2} \left|V_{ub}\right|^2 \left(\frac{2E_1 E_2}{2\pi m_B^2}\right) |\mathbf{p}|\, 2(m_l^2 + m_{\nu_l}^2) f_B^2 \end{aligned} \tag{33}$$

Noting that

$$\frac{2E_1 E_2 \, |\mathbf{p}|}{m_B^2} \approx \frac{1}{4} m_B$$

we get

$$\Gamma(B^- \to l^- \bar{\nu}_l) \approx \frac{G_F^2}{8\pi} \left|V_{ub}\right|^2 m_B m_l^2 f_B^2 \tag{34}$$

Thus we see that for this decay

$$S^2 + P^2 = 2(m_l^2 + m_{\nu_l}^2) f_B^2 \tag{35}$$

Hence we can parametrize $(S_f^2 + P_f^2)$ in terms of two form factors $F_V^{\Lambda_c - p}(s)$ and $F_A^{\Lambda_c - p}(s)$:

$$\begin{aligned} P_f^2 &= f_B^2 (m_{\Lambda_c} + m_p)^2 \left[\left(\frac{m_{\Lambda_c} - m_p}{m_{\Lambda_c} + m_p}\right) F_V^{\Lambda_c - p}(s)\right]^2_{s = m_B^2} \\ S_f^2 &= f_B^2 (m_{\Lambda_c} + m_p)^2 \left[F_A^{\Lambda_c - p}(s)\right]^2_{s = m_B^2} \end{aligned} \tag{36}$$

It is easy to see that for $F_V = 1$ and $F_A = 1$, it reduces to form of Eq.(35). Using the experimental values for the masses and $f_B \approx 180$ MeV, we get from Eq.(33)

$$(0.175)[F_V^{\Lambda_c - p}(m_B^2)]^2 + [F_A^{\Lambda_c - p}(m_B^2)]^2 = (3.1 \pm 1.1) \times 10^{-2} \tag{37}$$

The dominant contribution comes from the axial vector form factor. The decay $B_c^- \to n\bar{p}$ would give information for nucleon form factors:

$$\begin{aligned} P_f^2 &= f_{B_c}^2 (m_n + m_p)^2 \left[\frac{m_n - m_p}{m_n + m_p} F_V(s)\right]^2_{s = m_{B_c}^2} \approx 0 \\ S_f^2 &= f_{B_c}^2 (m_n + m_p)^2 \left[F_A^2(s)\right]_{s = m_{B_c}^2} \end{aligned} \tag{38}$$

The baryon decay modes of B -mesons also provide the means to explore the baryon form factors at high s. Finally, we note that Eq.(36), give the $SU(3)$ breaking factor $\xi = \frac{f_{B_s}}{f_B}$ in Eq.(23).

3 Time- Dependent Baryon Decay Modes of B_q^0

Define the amplitudes

$$\begin{aligned}
\mathcal{A}^{\lambda_1\lambda_2}(t) &= \frac{\left[\Gamma(B_q^0(t)\to f)-\Gamma(\bar{B}_q^0(t)\to\bar{f})\right]_{\lambda_1\lambda_2}+\left[\Gamma(B_q^0(t)\to\bar{f})-\Gamma(\bar{B}_q^0(t)\to f)\right]_{\lambda_1\lambda_2}}{\sum_{\lambda_1\lambda_2}\left[\Gamma(B_q^0(t)\to f,\bar{f})+\Gamma(\bar{B}_q^0(t)\to\bar{f},f)\right]_{\lambda_1\lambda_2}} \\
&= \frac{-2\sin\Delta mt\left[\operatorname{Im}e^{2i\phi_M}(M_f^*\bar{M}'_f+M'^*_f\bar{M}_{\bar{f}})\right]}{\sum_{\lambda_1\lambda_2}\left[\left|M_f^2\right|+\left|\bar{M}_{\bar{f}}^2\right|+\left|M'^2_{\bar{f}}\right|+\left|M'^2_f\right|\right]}
\end{aligned} \tag{39}$$

$$\begin{aligned}
\mathcal{F}^{\lambda_1\lambda_2}(t) &= \frac{\left[\Gamma(B_q^0(t)\to f)-\Gamma(\bar{B}_q^0(t)\to\bar{f})\right]_{\lambda_1\lambda_2}-\left[\Gamma(B_q^0(t)\to\bar{f})-\Gamma(\bar{B}_q^0(t)\to f)\right]_{\lambda_1\lambda_2}}{\sum_{\lambda_1\lambda_2}\left[\Gamma(B_q^0(t)\to f,\bar{f})+\Gamma(\bar{B}_q^0(t)\to\bar{f},f)\right]_{\lambda_1\lambda_2}} \\
&= \frac{\cos\Delta mt\left[\left|M_f^2\right|+\left|\bar{M}_{\bar{f}}^2\right|-\left|M'^2_{\bar{f}}\right|-\left|\bar{M}'^2_f\right|\right]-2\sin\Delta mt\left[\operatorname{Im}e^{2i\phi_M}(M_f^*\bar{M}'_f-M'^*_f\bar{M}_{\bar{f}})\right]}{\sum_{\lambda_1\lambda_2}\left[\left|M_f^2\right|+\left|\bar{M}_{\bar{f}}^2\right|+\left|M'^2_{\bar{f}}\right|+\left|M'^2_f\right|\right]}
\end{aligned} \tag{40}$$

Thus

$$\begin{aligned}
&8\left[(S_f^2+P_f^2)+r^2(S'^2_{\bar{f}}+P'^2_{\bar{f}})\right]\mathcal{A}^{\lambda_1\lambda_2}(t) \\
= \; &2\sin\Delta mt\left\{\begin{array}{l}\sin(2\phi_M-\gamma)\left[2r(1+\lambda_1\lambda_2)(S_fS'_{\bar{f}}\cos(\delta_s^f-\delta_s^{\bar{f}})+P_fP'_{\bar{f}}\cos(\delta_p^f-\delta_p^{\bar{f}})\right] \\ -\cos(2\phi_M-\gamma)\left[2r(\lambda_1+\lambda_2)(S_fP'_{\bar{f}}\sin(\delta_s^f-\delta_p^{\bar{f}})+S'_{\bar{f}}P_f\sin(\delta_p^f-\delta_s^{\bar{f}})\right]\end{array}\right\}
\end{aligned} \tag{41}$$

$$
8\begin{bmatrix} (S_f^2+P_f^2) \\ +r^2(S'^2_{\bar f}+P'^2_{\bar f}) \end{bmatrix}\mathcal{F}^{\lambda_1\lambda_2}(t) =
\begin{array}{l}
\cos\Delta m t\left\{\begin{array}{c} (S_f^2+P_f^2)\begin{bmatrix} 2(1+\lambda_1\lambda_2) \\ +(\alpha_f+\bar\alpha_{\bar f})(\lambda_1+\lambda_2) \end{bmatrix} \\ -r^2(S'^2_{\bar f}+P'^2_{\bar f})\begin{bmatrix} 2(1+\lambda_1\lambda_2) \\ +(\bar\alpha'_f+\alpha'_{\bar f})(\lambda_1+\lambda_2) \end{bmatrix} \end{array}\right\} \\
-2\sin\Delta m t\left\{\begin{array}{c} \cos(2\phi_M-\gamma)\begin{bmatrix} -2r(1+\lambda_1\lambda_2) \\ (S_f S'_{\bar f}\sin(\delta_s^f-\delta_s^{\bar f}) \\ +P_f P'_{\bar f}\sin(\delta_p^f-\delta_p^{\bar f})) \end{bmatrix} \\ +\sin(2\phi_M-\gamma)\begin{bmatrix} 2r(\lambda_1+\lambda_2) \\ (S_f P\cos(\delta_s^f-\delta_p^{\bar f}) \\ +P_f S'_{\bar f}\cos(\delta_p^f-\delta_s^{\bar f}) \end{bmatrix} \end{array}\right\}
\end{array}
\tag{42}
$$

These are general expressions for the time-dependent decay modes in the rest frame of B_q^0. From Eqs.(41) and (42), the even and odd time-dependent

decay amplitudes are given by

$$\begin{aligned}
\mathcal{A}(t) &\equiv (\mathcal{A}^{++}(t)+\mathcal{A}^{--}(t)) \\
&= \frac{2r\sin\Delta mt\sin(2\phi_M-\gamma)\left[S_f S'_{\bar f}\cos(\delta_s^f-\delta_s^{\bar f})+P_f P'_{\bar f}\cos(\delta_p^f-\delta_p^{\bar f})\right]}{(S_f^2+P_f^2)+r^2(S'^2_{\bar f}+P'^2_{\bar f})}
\end{aligned} \tag{43}$$

$$\begin{aligned}
\Delta\mathcal{A}(t) &\equiv \mathcal{A}^{++}(t)-\mathcal{A}^{--}(t) \\
&= \frac{-2r\sin\Delta mt\cos(2\phi_M-\gamma)\left[S_f P'_{\bar f}\sin(\delta_s^f-\delta_p^{\bar f})+S'_{\bar f}P_f\cos(\delta_p^f-\delta_s^{\bar f})\right]}{(S_f^2+P_f^2)+r^2(S'^2_{\bar f}+P'^2_{\bar f})}
\end{aligned} \tag{44}$$

$$\begin{aligned}
\mathcal{F}(t) &= \mathcal{F}^{++}(t)+\mathcal{F}^{--}(t) \\
&= \frac{\begin{array}{c}\cos\Delta mt\left[(S_f^2+P_f^2)-r^2(S'^2_{\bar f}+P'^2_{\bar f})\right]+2r\sin\Delta mt\cos(2\phi_M-\gamma) \\ \times\left[S_f S'_{\bar f}\sin(\delta_s^f-\delta_s^{\bar f})+P_f P'_f\sin(\delta_p^f-\delta_p^{\bar f})\right]\end{array}}{2\left[(S_f^2+P_f^2)+r^2(S'^2_{\bar f}+P'^2_{\bar f})\right]}
\end{aligned} \tag{45}$$

$$\begin{aligned}
\Delta\mathcal{F}(t) &\equiv \mathcal{F}^{++}(t)-\mathcal{F}^{--}(t) \\
&= \frac{\cos\Delta mt\left[(S_f^2+P_f^2)(\alpha_f+\bar\alpha_{\bar f})-r^2(S'^2_{\bar f}+P'^2_{\bar f})(\bar\alpha'_f+\alpha'_{\bar f})\right]}{2\left[(S_f^2+P_f^2)+r^2(S'^2_{\bar f}+P'^2_{\bar f})\right]} \\
&\quad -\frac{2r\sin\Delta mt\sin(2\phi_M-\gamma)\left[S_f P'_{\bar f}\cos(\delta_s^f-\delta_p^{\bar f})+P_f S'_{\bar f}\cos(\delta_p^f-\delta_s^{\bar f})\right]}{(S_f^2+P_f^2)+r^2(S'^2_{\bar f}+P'^2_{\bar f})}
\end{aligned} \tag{46}$$

For B_d^0, $r=-\lambda^2\sqrt{\bar\rho^2+\bar\eta^2}\approx-(0.02\pm0.006)$ [4], $\phi_M=-\beta$; for B_s^0, $r=-\sqrt{\bar\rho^2+\bar\eta^2}\approx-(0.40\pm0.13)$ [4], $\phi_M=0$. First term of Eq.(46) has an important implication: This term is zero, if $\alpha_f=-\bar\alpha_{\bar f}$; $\bar\alpha'_f=-\alpha'_{\bar f}$ as implied by CP-conservation. The finite value of this term would imply CP violation in baryon decay. The above equations are simplified if we assume the validity

of Eq.(24). In that case we have

$$\mathcal{A}(t) = \frac{2r\sin\Delta mt\sin(2\phi_M-\gamma)}{1+r^2} \tag{47}$$

$$\Delta\mathcal{A}(t) = 0 \tag{48}$$

$$\mathcal{F}(t) = \frac{1-r^2}{1+r^2}\cos\Delta mt \tag{49}$$

$$\begin{aligned}\Delta\mathcal{F}(t) &= \frac{1-r^2}{2(1+r^2)}(\alpha_f+\bar{\alpha}_{\bar{f}})\cos\Delta mt \\ &\quad -\frac{4r\sin\Delta mt\sin(2\phi_M-\gamma)S_fP_f}{(1+r^2)(S_f^2+P_f^2)}\end{aligned} \tag{50}$$

Eq.(47) gives a means to determine the weak phase $2\beta+\gamma$ or γ in the baryon decay modes of B_d^0 and B_s^0 respectively. Non-zero $\cos\Delta mt$ term in $\Delta\mathcal{F}(t)$ would give clear indication of CP violation especially for baryon decay modes of B_d^0, for which $r^2\leq 1$, so that $\frac{1-r^2}{1+r^2}\approx 1$. Assuming $CP-$invariance, we get from Eqs.(47) and (50)

$$\begin{aligned}-2S_fP_f &= (S_f^2+P_f^2)\frac{\Delta\mathcal{F}(t)}{\mathcal{A}(t)} \\ &= \{(1.04\pm 0.38)\times 10^{10}\text{MeV}^4]\frac{\Delta\mathcal{F}(t)}{\mathcal{A}(t)}\end{aligned} \tag{51}$$

The S_fP_f can be determined from the experimental value of $\frac{\Delta\mathcal{F}(t)}{\mathcal{A}(t)}$ in future experiments.

The baryon decay modes of B-mesons not only provide a means to test prediction of CP asymmetry viz $\alpha_f+\bar{\alpha}_{\bar{f}}=0$ for charmed baryons (discussed above) but also to test the CP-asymmetry in hyperon (antihyperon) decays viz absence of CP-odd observables $\Delta\Gamma, \Delta\alpha, \Delta\beta$ discussed in [3]. Consider for example the decays

$$B_q^0 \to p\bar{\Lambda}_c^- \to p\bar{p}K^0(p\bar{\Lambda}\pi^- \to p\bar{p}\pi^+\pi^-),$$

$$\bar{B}_q^0 \to \bar{p}\Lambda_c^+ \to \bar{p}p\bar{K}^0(\bar{p}\Lambda\pi^+ \to \bar{p}p\bar{b}\pi^-\pi^+)$$

By analyzing the fianl state $\bar{p}p\bar{K}^0, p\bar{p}K^0$, one may test $\alpha_f=-\bar{\alpha}_{\bar{f}}$ for the charmed hyperon. We note that for Λ_c^+, $c\tau=59.9\mu$m, whereas $c\tau=7.8$cm for $\Lambda-$hyperon [4], so that the decays of Λ_c^+ and Λ would not interfere with each other. By analysing the final state $\bar{p}p\pi^-\pi^+$ and $p\bar{p}\pi^+\pi^-$, one may

check CP-violation for hyperon decays. One may also note that for $(B_d^0, \bar{B}_d^0)$ complex, the competing channels viz $B_d^0 \to \bar{p}\Lambda_c^+$, $\bar{B}_d^0 \to p\bar{\Lambda}_c^-$ are doubly Cabibbo supressed by $r^2 = \lambda^4 \left(\bar{\rho}^2 + \bar{\eta}^2\right)$ unlike $(B_s^0 - \bar{B}_s^0)$ complex where the competing channels are supressed by a factor of $(\bar{\rho}^2 + \bar{\eta}^2)$. Hence $B_d^0(\ \bar{B}_d^0)$ decays are more suitable for this type of analysis. Other decays of intrest are

$$\begin{aligned}
B^- &\to \Lambda\bar{\Lambda}_c^- \to \Lambda\bar{\Lambda}\pi^- \to p\pi^-\bar{p}\pi^+\pi^- \\
B^+ &\to \bar{\Lambda}\Lambda_c^+ \to \bar{\Lambda}\Lambda\pi^+ \to \bar{p}\pi^+p\pi^-\pi^+ \\
B_c^- &\to \bar{p}\Lambda \to \bar{p}p\pi^- \\
B_c^+ &\to p\bar{\Lambda} \to p\bar{p}\pi^+
\end{aligned}$$

The non-leptonic hyperon (antihyperon) decays $N \to N'\pi(\bar{N} \to \bar{N}'\bar{\pi})$ are related to each other by CPT

$$\begin{aligned}
a_l(I) &= \left\langle f_{lI}^{out} \left| H_W \right| N \right\rangle = \eta_f e^{2i\delta_l(I)} \left\langle \bar{f}_{lI}^{out} \left| H_W \right| \bar{N} \right\rangle \\
&= \eta_f e^{2i\delta_l(I)} \bar{a}_l^*(I)
\end{aligned}$$

Hence

$$\bar{a}_l(I) = \eta_f e^{2i\delta_l(I)} \bar{a}_l^*(I) = (-1)^{l+1} e^{i\delta_l(I)} e^{-i\phi} \left| a_l \right|$$

where we selected the phase $\eta_f = (-1)^{l+1}$. Here I is the isospin of the final state and ϕ is the weak phase. Thus necessary condition for non-zero CP odd observables is that the weak phase for each partial wave amplitude should be different [see ref [3] for details; for a review see first ref in [1]].

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Figure Captions

Figure1a: W-exchange diagram for $B_q^0 \to N_1\bar{N}_2\,(M_f)$

Figure1b: W-exchange diagram for $\bar{B}_q^0 \to N_1\bar{N}_2\left(\bar{M}_f'\right)$

Figure2: Annihilation diagram for $B^- \to N_1\bar{N}_2$